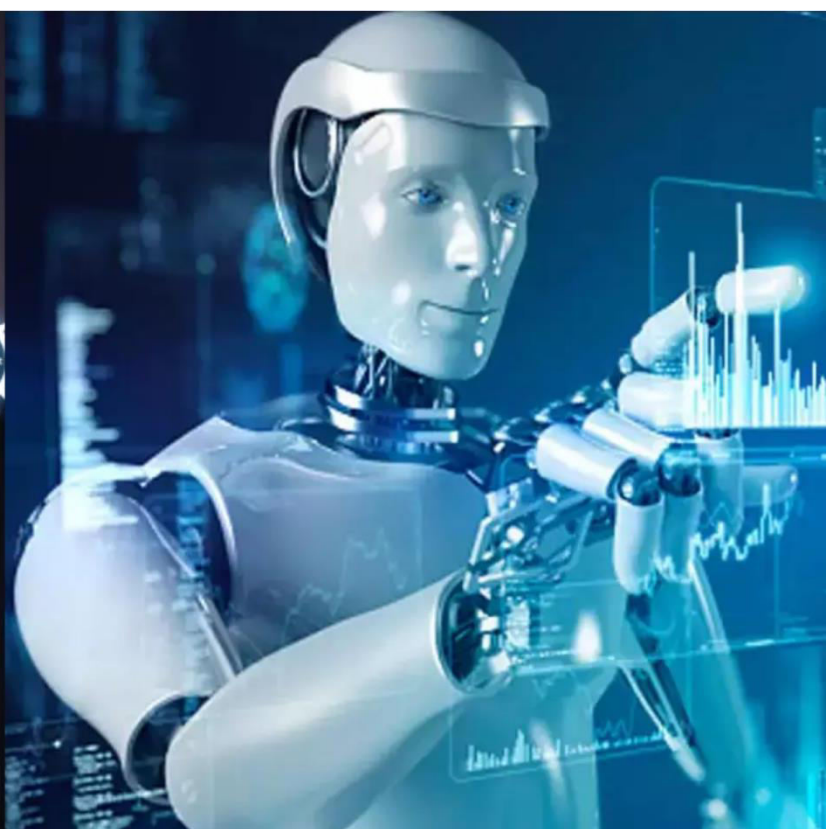




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A Review Paper on the Evolution of Integral Transform

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ABSTRACT: Integral transforms have become essential tools in mathematics, signal processing, image analysis, and engineering applications. This study reviews the development of the Fourier Transform (FT), Fractional Fourier Transform (FrFT), Simplified Fractional Fourier Transform (SFrFT), Mellin Transform, and Fourier–Mellin Transform (FMT), highlighting their theoretical foundations and practical significance. The literature shows that these transforms have been successfully applied in areas such as signal processing, image registration, pattern recognition, optical systems, quantum mechanics, and IoT security. The Simplified Fractional Fourier–Mellin Transform (SFrFMT) is a promising framework that combines the advantages of the Fourier, fractional Fourier, and Mellin transforms, offering improved computational efficiency and flexibility.

KEYWORDS: Fourier Transform (FT), Fractional Fourier Transform (FrFT), Simplified Fractional Fourier Transform (SFrFT), Mellin Transform, signal processing, pattern recognition, quantum mechanics.

I. INTRODUCTION

The development of integral transforms has played a crucial role in advancing mathematical analysis, signal and image processing and engineering applications. Among these transforms, the Fourier Transform (FT) was introduced by Joseph Fourier in 1807. It is one of the most fundamental and widely used mathematical tools.

The Fourier transform is a generalization of the Fourier series [5]. It only applies to continuous and periodic functions. We defined the Fourier Transform of a piecewise continuous and absolutely integrable function $x(t)$ is given by [2]

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt,$$

where $K(t, \omega) = e^{-j\omega t}$ is the kernel of the transform.

Its ability to represent complex functions as combinations of sinusoidal components has made it indispensable in areas such as communication systems, digital signal processing, medical imaging, quantum mechanics, circuit analysis, and wireless networks [1]-[4]. The mathematical foundation of many modern transform techniques is rooted in the theory of generalized integral transforms, which provides a unified framework for analyzing functions, distributions, and operational methods [27]. To address the limitations of the Classical Fourier Transform for analyzing non-stationary signals, researchers introduced the Fractional Fourier Transform (FrFT), which generalizes the FT by providing intermediate domains between time and frequency representations [6, 7].

The fractional Fourier transform is given from the point of view of the linear integral transform, which reflects the most basic properties of the fractional Fourier transform and is the most rigorous form of mathematical definition of the fractional Fourier transform and it is given below [26]:

$$g(u) = F^p\{f(x)\} = \int_{-\infty}^{\infty} f(x) K_p(u, x) dx,$$

where K_p is the FrFT kernel function defined as follows:

$$K_p(u, x) = \begin{cases} \sqrt{\frac{1-j \cot a}{2\pi}} \exp \left[j \left(\frac{x^2 + u^2}{2 \tan a} - \frac{xu}{\sin a} \right) \right] & a \neq n\pi \\ \delta(x-u) & a \neq 2n\pi \\ \delta(x+u) & a = (2n \pm 1)\pi \end{cases}$$

Is the formula $a = p\frac{\pi}{2}, p \neq 2n$. The order p is taken with 4 as the least positive period. $p = 0$ is the original; $p = 1$ is the FT, and when p is transformed between $[0, 1]$. It is called FrFT [7].

Over the years, significant theoretical and practical developments have been made in fractional transforms, including generalized fractional transforms, operational calculus, and digital implementations, leading to enhanced performance in signal analysis, optical systems, image processing and quantum mechanics applications [8]-[10]. Furthermore, the Simplified Fractional Fourier Transform (SFrFT) was developed as a computationally efficient alternative to the conventional FrFT while retaining its essential capabilities, making it suitable for practical engineering and optical applications [11, 12, 13].

The Simplified fractional Fourier transform with parameter α of $f(t)$ denoted $O_{F(1)}^\alpha(f(t))$ performs a linear operation given by the integral transform [12],

$$O_{F(1)}^\alpha(f(t)) = F_\alpha\{f(t)\}(u) = \int_{-\infty}^{\infty} f(t) K_\alpha(t, u) dt,$$

where the kernel, $K_\alpha(t, u) = (j2\pi)^{-1/2} e^{(-jut + \frac{j}{2}t^2 \cot \alpha)}$

Parallel to these developments, the Mellin transform emerged as a powerful mathematical framework for analyzing scale-invariant phenomena, asymptotic behavior, and multiplicative convolution problems [16, 17]. Some variants of Mellin like β -Mellin [18], etc.

Let $h(t)$ be the function defined on the interval $(0, \infty)$, the Mellin transform of the function $h(t)$, denoted by $H(u)$ is as follows [17],

$$H(u) = \mathcal{M}\{h(t); u\} = \int_0^\infty h(t) t^{u-1} dt,$$

where u is complex.

The integration of Fourier and Mellin transform concepts gave rise to the Fourier-Mellin Transform (FMT), which exhibits desirable properties such as translation, rotation, and scale invariance, making it highly effective for image registration, pattern recognition, image matching, watermarking, and image stabilization tasks [21, 22]. Further extensions involving Fractional Fourier-Mellin Transforms and their generalized forms have demonstrated significant potential in solving differential equations, signal processing problems, optical beam propagation, radar systems, and image analysis applications [23, 24].

Recent advancements in image processing and security-sensitive applications, particularly within Internet of Things (IoT) environments, have further highlighted the importance of transform-based techniques for robust feature extraction, privacy preservation, and secure information processing [25]. Consequently, the study of the Simplified Fractional Fourier-Mellin Transform (SFrFMT) represents a natural progression in the evolution of integral transforms, combining the advantages of Fourier, Fractional Fourier and Mellin transforms in a unified framework with broad applicability across modern scientific and engineering disciplines [11], [23]-[27]. These articles describe the development of transforms and their theoretical foundations and practical significance. This is distributed in some sections as a review of Fourier transform, fractional Fourier transform, simplified fractional Fourier transform, Mellin transform and some special transforms.

II. REVIEW OF FOURIER TRANSFORM

Nisha Chittora, Deepika Babel, "A Brief Study on Fourier Transform and Its Applications". The Fourier Transform is a powerful mathematical tool that simplifies complex problems by representing signals as sums of complex exponentials, making it both efficient and widely applicable across science and engineering. It plays a central role in signal processing and digital image processing, where images can be analyzed as two-dimensional waveforms, enabling filtering, encoding, and transformation. Its influence extends into quantum mechanics, mechanical systems, industrial processes, wireless networks, and power distribution systems, where it helps mitigate disturbances with high accuracy and noise immunity. Beyond engineering, the Fourier Transform has significant applications in medical science, such as imaging and diagnostics, and in everyday technology like cell phones, where it underpins the signal processing methods that make mobile communication possible. By offering faster, more accurate, and elegant solutions, the Fourier Transform remains indispensable for tackling modern challenges across diverse fields. This article includes the definition of the Fourier transform and highlights its some properties. It discusses about four different forms of the Fourier transform and reviews applications of the Fourier transform in Medical Engineering, Cell phones and their roles.[1].

Prof. Poonam Rajvardhan Patil, Prof. Shankar Akaram Patil, "Applications of Fourier Transform in Engineering Field". This article explores the Fourier Transform in depth, covering its definition, inverse form, and discrete version along with their properties, while emphasizing that the Discrete Fourier Transform is the most suitable representation for digital computer applications. It highlights how Fourier methods provide elegant solutions to complex problems, particularly in solving partial differential equations and initial boundary value problems, where the transform reduces the number of independent variables by one, simplifying calculations significantly. The study also demonstrates the wide-ranging applications of Fourier analysis in circuit design, signal processing, image processing, linear time-invariant (LTI) systems, and even in everyday technologies like cell phones, where it underpins efficient communication [2].

Shana Anjum, Prof. Vaishali Sahu, "Fundamentals and Literature Review of Fourier Transform". This paper examines the fundamentals of the Fourier Transform, Fourier Series, and the Digital Fourier Transform, along with their properties and reviews, showing how these mathematical tools provide powerful ways to represent and analyze signals. The Fourier Transform itself breaks down a waveform or signal into an alternate form expressed through sines and cosines, revealing that any waveform can be reconstructed as a sum of sinusoidal functions. Closely related, the Fourier Series expands a periodic function into an infinite sum of sines and cosines, relying on the orthogonality of these functions to simplify complex periodic behavior [3].

Mrs. Mamta Kansal, "A Review of Research Papers on Fourier Transforms & Statistical Fourier Analysis". This paper explores the fundamental role of Fourier analysis in statistical time-series studies, examining the effects of the Fourier hypothesis and the properties of the limited Fourier transform and Fourier arrangements. It discusses advanced concepts such as harmonic indices, canonical Fourier methods, and statistical Fourier analysis, while emphasizing that the Fourier series forms the foundation of Fourier analysis by expressing functions as unbounded expansions of sines and cosines. This approach is especially valuable in physics and engineering, where periodic functions can be more easily manipulated through such expansions. Fields like electronics, quantum mechanics, and electrodynamics rely heavily on Fourier series for practical applications, and the paper also introduces a method for performing the Fractional Fourier Transform in digital domains, highlighting its versatility and importance in both theoretical and applied contexts [4].

Mr. Karan Asher, "Fourier series And Fourier Transform". This paper provides an accessible introduction to Fourier Series and Fourier Transform, explaining both their meaning and their significance in applied sciences. The Fourier Series is presented as a way to represent periodic functions through sums of sines and cosines, with details on its general form, coefficient evaluation, and key properties. Building on this, the Fourier Transform is described as a powerful tool for converting signals from the time domain into their frequency domain equivalents, with the Discrete Fourier Transform offering a practical method to handle continuous signals in digital form. Beyond theory, the paper highlights real-world applications of Fourier analysis in areas such as signal processing, digital filtering, image processing, heat conduction, and wave propagation, emphasizing that its influence extends across nearly every branch of applied science. In essence, the discussion captures both the mathematical foundations and the immense practical potential of Fourier analysis, showing how it bridges abstract concepts with everyday technological solutions [5].

III. REVIEW OF FRACTIONAL FOURIER TRANSFORM

Victor Namias, "The Fractional Order Fourier Transform and its Application to Quantum Mechanics". This article introduces the concept of fractional-order Fourier transforms, extending the ordinary Fourier transform of order 1 into a broader framework with integral representations that allow the construction of tables for fractional cases. By developing a generalized operational calculus, parallel to the familiar one used for standard transforms, the paper presents a powerful technique for solving classes of ordinary and partial differential equations of order $n > 2$, particularly those arising in quantum mechanics from quadratic Hamiltonians. It applied rule of the generalized operational calculus to the solution of the time-independent Schrödinger equation for the harmonic oscillator and for free electrons in a time-varying uniform magnetic field. The method is illustrated through applications to both free and forced quantum harmonic oscillators, where closed-form Green's functions are derived and then extended to three-dimensional problems such as the motion of electrons in a constant magnetic field, yielding stationary states, energy levels, and wave packet evolution. Finally, the approach is applied to second-order partial differential equations with time-dependent coefficients, offering a systematic way to describe the quantum dynamics of electrons in varying magnetic fields [6].

Yilin Wang, "Fractional Fourier Transform and its application". This paper introduces both the Fourier Transform (FT) and its extension, the Fractional Fourier Transform (FrFT), highlighting their definitions, differences, and applications. The FT is a classical linear transformation that converts functions between the time and frequency domains using an orthogonal basis of sines and cosines, while the FrFT generalizes this idea by allowing transformations of fractional order. By transforming an image function into a series of periodic functions, the FrFT provides deeper insight into non-smooth signals, nonlinear systems, and complex phenomena. The article illustrates how this powerful tool can be applied to digital image processing and extends its significance to broader fields such as signal processing, communication, optical imaging, quantum information and face recognition technology [7].

Bellamkonda Sivadas and Dr. Dipti Vasdev, "A Review of Fractional Transforms and their Practical Applications". This review article explores the fundamentals and practical applications of fractional transforms, powerful mathematical tools that extend classical integral transforms to non-integer orders, thereby broadening their scope for analyzing signals and functions. It traces the background and development of fractional transforms and provides a comprehensive study of various types including Riemann-Liouville, Caputo, Laplace, Fourier, and others along with their properties and practical utility. Beyond theory, the paper emphasizes how fractional transforms are applied in diverse fields such as finance, physics, image analysis, and signal processing, while also highlighting emerging trends that showcase their growing relevance in modern research and technology. It gives some open questions to continuously changing scientific research like explores digital technology, genetic databases, and many others [8].

Ahmed I. Zayed, "On the Relationship between the Fourier and Fractional Fourier Transforms". This article examines how small modifications to the classical Fourier Transform, such as varying angle values, lead to the formation of the Fractional Fourier Transform. It explains that many of the properties of the FrFT like its inversion formula and sampling theorems can be derived directly from those of the standard Fourier Transform through simple changes of variables. By building on the familiar framework of the classical transform, the paper shows how fractional extensions preserve core principles while offering new flexibility and broader applications in mathematical analysis and signal processing [9].

Imtiaz Waheed, Mujeeb Ur Rehman, "On the fractional Fourier transforms with respect to functions and their applications". This article introduces the concept of Φ -fractional Fourier transforms within the framework of fractional calculus, extending the classical Fourier approach to a generalized form with respect to a function and its inverse. It explores fractional integrals, derivatives, and other transformation methods that operate within or alongside functions, establishing clear connections between the fractional Fourier transform and differential as well as integral operators. The study also investigates the inverse operator, key properties of this transform, and the convolution operation, along with its associated rules and operational characteristics. Finally, the paper demonstrates practical applications of the generalized fractional Fourier transform, including solving partial and ordinary differential equations of fractional order, and specifically addressing the time-fractional Riemann-Liouville linear inhomogeneous fractional evolution equation, illustrating its broad utility in advanced mathematical and physical contexts [10].

IV. REVIEW OF SIMPLIFIED FRACTIONAL FOURIER TRANSFORM

Soo-Chang Pei and Jian-Jiun Ding, "Simplified fractional Fourier transforms". This article explores the long-standing use of the Fractional Fourier Transform (FRFT) and its importance in practical applications, particularly through fractional filters such as chirp noise removal and the fractional Hilbert transform and fractional correlation techniques like scaled space-variant pattern recognition. It introduces several simplified versions of the FRFT, known as Simplified Fractional Fourier Transforms (SFRFTs), which are special cases of the linear canonical transform. These simplified forms retain the same capabilities as the original FRFT for filter design and correlation but are easier to implement in digital computation, optical systems, gradient-index media, and radar applications. The study focuses on identifying the simplest transform that can match the power of the FRFT, discussing both the formulas and properties of SFRFTs as well as their practical implementations. Although SFRFTs may lack certain additive properties, they remain highly effective and offer strong potential to replace the original FRFT in many real-world applications [11].

V. D. Sharma, "Introduction to Generalized Simplified Fractional Fourier Transform (SFRFT)". This article examines three types of simplified fractional Fourier transforms, focusing on their testing functions and the distribution of type I SFRFT. It also introduces the concept of generalized simplified fractional Fourier transforms, exploring how these can be converted into simplified Fourier transforms and analyzing their properties. By studying these variations, the paper highlights both the theoretical foundations and practical aspects of simplified fractional transforms, showing how they extend the utility of the classical Fourier framework while offering more efficient approaches for computation and application [12].

Soo-Chang Pei, Jian-Jiun Ding, "Fractional, Canonical, And Simplified Fractional Cosine Transforms". This article explores how the Fourier Transform can be generalized into forms like the Fractional Fourier Transform (FRFT), Linear Canonical Transform (LCT), and Simplified Fractional Fourier Transform (SFRFT), each extending the utility of the original transform to solve problems that traditional methods struggle with. Building on this, it introduces the Fractional Cosine Transform (FRCT), Canonical Cosine Transform (CCT), and Simplified Fractional Cosine Transform (SFRCT), which mirror the FRFT, LCT, and SFRFT but are especially efficient for handling even real functions. Their applications include optical system analysis, space-variant pattern recognition, and digital implementations, where they significantly reduce computational effort. For example, FRCT and CCT can replace FRFT and LCT when inputs are even functions, saving about half the real number multiplications, while SFRCT can replace them for real, even functions, cutting computation by nearly three-quarters. Overall, these cosine-based transforms provide a streamlined, powerful alternative for practical applications [13].

Ervin Sejdić, Igor Djurović and LJubiša Stanković, "Fractional Fourier transform as a signal processing tool: An overview of recent developments". This article provides an overview of recent contributions of the fractional Fourier Transform (FRFT) in various fields. It is particularly geared toward signal processing practitioners, emphasizing practical digital realizations of the FRFT. The discussion is organized into three main areas: the relationship between FRFT and other mathematical transforms, approaches for practical realization of the FRFT, and its practical applications. From these explorations, the paper highlights how FRFT is closely connected to time-frequency and linear canonical transforms, while also noting that many digital realizations still lack some of the properties of the continuous version. Overall, the FRFT emerges as a valuable tool in signal processing, offering significant advantages over traditional Fourier analysis, with its practical applications like filtering, watermarking, radar applications, communications, compression, pattern recognition, cryptography, Fractal signal processing and many more as expected to expand rapidly in the years ahead [14].

Vidya Sharma and P. R. Langade, "Simplified Fractional Fourier Transform and its Operators". In 1807, Joseph Fourier introduced the idea of the Fourier Transform, which later evolved into more advanced forms such as the Fractional Fourier Transform. Building on this, Namias applied the concept to solve various Schrödinger equations, demonstrating how complex fractional transforms could be simplified. These simplifications gave rise to the Simplified Fractional Fourier Transform (SFRFT), a more practical version that retains the power of the original while being easier to use. The article explores generalization of SFRFT and outlines its key properties, including differentiation, linearity, shifting, scaling, and modulation [15].

V. REVIEW OF MELLIN TRANSFORM

Teshayeva Gulhayo Farhod kizi, "The Mellin Integral Transform and Its Applications". This article explores the theoretical foundations of the Mellin transform, outlining its core properties and connections to other classical integral transforms. It explains how the Mellin transform is particularly effective in addressing problems involving scale invariance, asymptotic behavior through residues and contour integration, and multiplicative convolution. The study shows that the Mellin transform provides a powerful analytical framework for evaluating improper integrals, analyzing special functions such as Gamma and Beta functions, and simplifying complex mathematical expressions. Beyond theory, it highlights practical applications in areas like mathematical physics, signal and image processing, and algorithm analysis, confirming that the Mellin transform is both versatile and impactful. Ultimately, the paper demonstrates how this integral transform bridges theoretical and applied mathematics, offering wide potential for future research and innovation [16].

Maliha Rashid, Amna Kalsoom, Maria Sager, Mustafa Inc*, Dumitru Baleanu and Ali S. Alshomrani, "Mellin transform for fractional integrals with general analytic kernel". This article examines the classification of fractional integrals and derivatives into two main categories: those with and without a general analytic kernel, a concept introduced in 2019. It proposes several operators of fractional calculus within broader classes of operators and defines the Mellin transform for fractional differential operators with general analytic kernels, considering both Riemann-Liouville and Caputo derivatives of order $\varsigma \geq 0$ with a fixed parameter ϱ . The study further establishes connections between the Mellin transform and other classical transforms such as Laplace and Fourier, highlighting how these relationships enrich the analytical framework of fractional calculus [17].

Antonio De Sena and Davide Rocchesso, "A Fast Mellin and Scale Transform". This article introduces a fast algorithm for the discrete-scale and β -Mellin transforms, designed to approximate the continuous-time transform in a discrete-time, discrete-scale setting with sub-quadratic complexity. Built on the established relationship between Mellin and Fourier transforms, the algorithm is both practical and accurate. The paper first lays out the theoretical background of Mellin, β -Mellin, and scale transforms, then presents the algorithm itself, analyzing its computational efficiency and precision. It also explores how different interpolation methods affect the algorithm's performance, making the study a valuable contribution to advancing efficient and reliable transform techniques in signal processing and mathematical analysis [18].

Enes Ata, I. Onur Kiyamaz, "New generalized Mellin transform and applications to partial and fractional differential equations". This article introduces the new generalized Mellin transform, refining and rebuilding earlier versions found in the literature while exploring its properties in depth. It highlights how this new approach can be applied across diverse fields such as number theory, combinatorics, and probability theory and demonstrates its power by deriving elementary functions and solving both partial and fractional differential equations. It studies the relationship between generalized Mellin transforms in the literature and the new generalized Mellin transform. This paper provided various examples of the new integral transformation applied to elementary functions. It shows a wide range of applications in the field of differential equations [19].

Adem Kılıçman* and Maryam Omran, "Note on fractional Mellin transform and applications". Flajolet et al. (1995) note that Hjalmar Mellin (1854–1933) gave his name to the Mellin transform, a powerful tool that links a function defined over the positive reals. Building on this foundation, the study explores the fractional Mellin transform by employing the Riemann–Liouville fractional integral operator and the Caputo fractional derivative of order $\alpha \geq 0$. It highlights key properties of the Mellin transform within the framework of fractional calculus and points to practical applications. Ultimately, the work encourages deeper investigation into the Mellin integral transform and its inverse, particularly in solving fractional equations with variable coefficients, opening pathways for broader mathematical and applied research [20].

VI. REVIEW OF SOME SPECIAL TRANSFORMS

J. R. Martínez-de Dios y A. Ollero, "A Real-Time Image Stabilization System Based on Fourier-Mellin Transform". This article presents a principle for real-time image stabilization using the Fourier-Mellin transform, which corrects both translation and rotation in images. To enhance robustness and accuracy, it discusses practical optimizations for image matching, implemented through two approaches: leveraging the symmetric properties of real data and applying

twiddle factors from the Cooley-Tukey algorithm. Extensive experiments across different applications demonstrate the system's high reliability and precision. Impressively, the method achieves real-time performance capturing, stabilizing, and displaying images at standard video rates (PAL and NTSC) on a basic Pentium III running at 800 MHz, without requiring specialized hardware or a dedicated software platform [21].

B Dasgupta, B N Chatterji, "Fourier-Mellin Transform based Image Matching Algorithm". This article explores different algorithms related to the Fourier-Mellin transform, focusing on the calculation of Fourier-Mellin descriptors as well as segmentation and feature-based matching techniques. It highlights the transform's key properties of translation, rotation, and scale invariance, which make it powerful for image analysis. The paper outlines the algorithm for computing Fourier-Mellin descriptors in five clear steps, ensuring a structured approach to the process. Additionally, it presents a segmentation and feature-based matching algorithm that determines lower and upper thresholds and other values needed to interpret images effectively. The study also considers the time complexity of these algorithms, offering insights into both their theoretical foundations and practical efficiency [22].

V. A. Sharma*, P. B. Deshmukh, "Analyticity of Two Dimensional Fractional Fourier-Mellin Transform". This article highlights the wide-ranging applications of the Fractional Fourier Transform (FRFT), from solving differential equations and modelling optical beam propagation to signal detection and correlation, where it effectively diagonalizes linear time-invariant operators central to signal processing. It also emphasizes the Mellin transform's versatility, with uses in navigation, radar systems, stress distribution analysis in infinite wedges, and even digital audio effects. Building on these foundations, the paper generalizes the two-dimensional fractional Fourier-Mellin transform in the distributional sense and establishes an analyticity theorem, showing how these advanced transforms expand both theoretical understanding and practical problem-solving across diverse scientific and engineering fields [23].

V. D. Sharma, P. B. Deshmukh, "Operational calculus for two dimensional fractional Fourier-Mellin transform". This article emphasizes how integral transforms simplify complex problems, with the Fourier-Mellin transform proving especially useful in diverse applications such as image registration, watermark detection, invariant pattern recognition, medical image alignment, plant leaf comparison, grayscale image reconstruction, and human face detection—even when scaling or rotation is involved. The work focuses on the role of shifting, scaling, and combined shifting-scaling operators, while also presenting several theorems related to differential operators. Together, these discussions highlight both the theoretical depth and practical versatility of the Fourier-Mellin framework in solving real-world challenges [24].

A. M. Al-Ghaili et al., "Review on Role of Image Processing Techniques to Enhancing Security of IoT Applications". This article addresses a critical challenge in image-based Internet of Things (IoT) applications: ensuring the security and privacy of processed information before it is shared with other systems. It surveys recent research on how image processing techniques are being applied to strengthen IoT security, covering methods that enhance privacy, safety, and robustness in applications such as medical imaging, surveillance, and smart devices. By reviewing approaches like secure feature extraction, watermarking, and privacy-preserving image analysis, the paper provides insights into the role of image processing in safeguarding IoT data. To guide researchers and practitioners, it also presents a comprehensive graphical framework that maps suitable image processing techniques to specific IoT security needs, offering a clear view of how these methods can be leveraged to better protect sensitive information in real-world applications [25].

VII. CONCLUSION

This literature review highlights the remarkable development of integral transforms, from the classical Fourier Transform to advanced techniques such as the Fractional Fourier Transform, Simplified Fractional Fourier Transform, Mellin Transform, and Fourier–Mellin Transform. The reviewed studies demonstrate their wide-ranging applications in signal processing, image analysis, communication systems, optics, and mathematical modelling. As research has progressed, these transforms have become more efficient and versatile in handling complex real-world problems. Among them, the Simplified Fractional Fourier–Mellin Transform (SFrFMT) stands out as a promising approach that combines the strengths of its predecessors while offering greater flexibility and computational efficiency, making it a valuable tool for future research and technological advancements.

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